ADVANCED MACRO – EXAM QUESTIONS -- Normal Period

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QUESTION 1 (5 points)

Assume that utility is separable in consumption and real money balances i.e., $u(c_t, m_t) = w(c_t) + v(m_t)$. Further assume that $v(\mathrm{m_t}) = \mathrm{m_t}[\mathrm{B} - \mathrm{D}\ln(\mathrm{m_t})]$, where B and D are positive parameters, and $w(c_t) = \ln c_t$.

a) Show that the demand for money takes a similar form to a classical money demand equation, $m_t = Ae^{-\alpha_t i_t}$, where $\mathrm{A} = e^{B/D-1}$ and $\alpha_t = 1/\mathrm{c_t D}$.

b) Show that there is a Laffer curve for seignorage. Tip: Recall (from Ljungqivst and Sargent, 2004, chapter 24) that steady state seniorage $\overline{s}$ is given by the product of the money base and the inflation rate and that the latter will equal the growth of money supply in steady state.

c) What is the rate of money growth that maximizes steady state seignorage revenues and how it relates to the interest rate elasticity of money demand?

d) In a few lines, discuss the macroeconomic logic for the existence of a Laffer curve for seignorage and the limits it imposes on monetary and fiscal policy depending on its shape.

e) From the fiscal theory of the price level we have the following relationship between the level of the real government debt (D/P) and the present value of tax and seignorage revenues minus government spending:

$$\frac{D_t}{P_t} = \sum_{i=0}^{\infty} \lambda_{t,t+i}[\tau_{t+i} + \overline{s}_{t+i} - g_{t+i}^P] \ ,$$

where λ is the discount factor, g is primary government spending, τ is tax revenue, and the outstanding nominal liabilities D_t are pre-determined at the beginning of the period, so the key variable to adjust is $\mathrm{P_t}$. Consider the alternative definition of seignorage as $\overline{s}_t = m_t \frac{i_t}{1+i_t}$. To simplify the algebra, re-write the money term in utility as

$v(m_t) = b\ln m_t$. Show that $\bar{s}_t = bc_t$ and that the price level is independent of the nominal money supply as long as $\tau_t - g_t + bc_t$ is independent of M_t. **(1.0 point)**

Solution:

a) Using FOC (and in analogy with the utility on consumption and labor where W/P is the opportunity cost of leisure), FOC deliver: $\frac{u_m(c_t,m_t)}{u_c(c_t,m_t)} = \frac{i}{1+i} \simeq i$.

But from the given functional form, we have:

$$u_m = B - D - D\ln(m_t)$$
$$u_c = w'(c_t) = 1/c_t$$

So,

$$\frac{B - D - D\ln(m_t)}{w'(c_t)} = i_t$$
$$B - D - i_t w'(c_t) = D\ln(m_t)$$
$$\frac{B}{D} - 1 - \frac{i_t w'(c_t)}{D} = \ln(m_t)$$

But recall that $\ln(1+x) \approx x$. Call x=B/D-1, so B/D-1=ln(B/D), so:

$$\ln\left(\frac{B}{D}\right) - \frac{i_t w'(c_t)}{D} = \ln(m_t)$$
$$m_t = \frac{B}{D}\exp\left[-\frac{1}{Dc_t}i_t\right] = A\exp(-\alpha_t i_t)$$

b) Seignorage in steady-state:

$$\bar{s} = \pi A\exp(-\alpha i) = \pi A\exp[-\alpha(r+\pi)]$$

.:

$$\frac{\partial \bar{s}}{\partial \pi} = A\exp[-\alpha(r+\pi)] - \pi\alpha A\exp[-\alpha(r+\pi)]$$
$$= A(1-\pi\alpha)\exp[-\alpha(r+\pi)]$$

So, seignorage is positive (but at a decreasing rate) for not so high values of π, in particular α π<=1, and negative otherwise. So there is a "Laffer curve" for seignorage.

c) Seignorage is maximized when the above expression = 0, i.e., α π=1. Since in steady state money growth= $\dot{m}$=inflation, then $\dot{m} = 1/\alpha = Dc^{ss}$.

d) From (a) and the given functional forms, we have: $\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = \frac{bc_t}{m_t} = \frac{i}{1+i} \therefore \frac{(1+i)bc_t}{i} = m_t$

From the given definition of seignorage, it then follows that:

$\overline{s} = \frac{i}{1+i} m_t = \frac{i}{1+i} \frac{(1+i)bc_t}{i} = bc_t$.

Now substitute this expression to substitute out $\overline{s}$ into the equation of the fiscal price theory to obtain:

$$\frac{D_t}{P_t} = \sum_{i=0}^{\infty} \lambda_{t,t+i} [\tau_{t+i} + bc_{t+i} - g_{t+i}^P]$$

As long as $\tau + bc + g$ is independent of m, so is P.

QUESTION 2 (2 points)

Use the risk-adjusted uncovered interest parity ("UIP") equation to describe the macroeconomic logic behind the Mundel's Trillema and discuss the constraints it can impose on domestic monetary policy.

Then, suppose you are a researcher having at your disposal historical data series on interest rates, exchange rates, and degree of capital controls in various countries, and knowing that capital controls and exchange rates have been far more rigid during certain periods and in certain countries than others (e.g. the period from World War II to about the 1970s vs. the period from the mid-1990s to today; China vs. the U.S. in the past 20 years). What econometric testing methodology, i.e. what kind of regression and statistical tests would you run to try to prove or reject the existence of a Mundellian Trillema?

Solution:

The risk-adjusted UIP below shows that, for given the external interest rate i* and level of capital control (or sovereign risk) ζ , if a country (central bank) moves the domestic interest rate i, then that will create expectation of a change in the exchange rate. So, if you keep the freedom of moving i, you have to give up the idea of a peg. Conversely if you peg the exchange rate, again with i* and ζ fixed, you automatically fix i, so you cannot have an autonomous monetary policy.

$$E\Delta e_t = i_t - i_t * + \zeta_t$$

A simple test would be to regress i on i* for countries/periods where and when the exchange rate is fixed and there is free capital mobility, and test whether the coefficient beta, as in the regression below is close to 1:

$$i = \alpha + \beta i * + \varepsilon_t$$

Conversely, running the same regressions for periods/countries when and where the exchange is floating and capital is fully mobile, should yield a coefficient closer to zero.

Similarly, for countries/periods where and when capital controls are stringent and assuming the exchange rate is a random walk, one would expect a beta closer to zero, as the domestic economy can de-link itself from monetary policy abroad.